Architecture and Music in the Music of Iannis Xenakis:

An analysis of *Metastaseis* for orchestra

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Music and mathematics always have been in a close relationship with each other. Music theory, harmonic and contrapuntal rules are to some extent based on numbers and formulas. French-Greek composer, architect and engineer, Iannis Xenakis (1922 – 2001) took this even further. He was interested in composing music based on complex mathematic formulas and designing architectural realizations of that music, or composing architecture-inspired music, a musical realization of architectural designs.[1] Due to the difficulty of analyzing Xenakis's music, there have been very few books written on his music that analyze the formulized structure of his pieces. However, his own book, *Formulized Music*, is still the best reference for studying his thoughts and ideas.[2]

In this article, after discussing Xenakis's background as a Greek architect who studied composition with Olivier Messiaen, I will discuss one of his ambitious projects, the architecture design of the Philips Pavilion in Expo 58, and then give a detailed analysis that traces the serial and stochastic elements of his orchestral piece *Metastaseis* (1953–54) which was composed based on the architectural design of that Philips Pavilion.

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[1] Paul Griffiths, "Iannis Xenakis, Composer Who Built Music on Mathematics, Is Dead at 78," *The New York Times*, Feb. 5, 2001, B7.
[2] Iannis Xenakis, *Formalized Music: Thought and Mathematics in Composition* (Stuyvesant, NY: Pendragon Press, 1992).

Background with Architecture and Music

Xenakis was born in Romania in 1922. He studied engineering and architecture in university but at the same time he took fundamental courses in music. After he graduated from Athens Technical University in Civil Architecture, Xenakis immigrated to Paris and resided there for the rest of his life.[3] He was employed at Le Corbusier's architecture studio in Paris.[4]

He was perceived as a talented architect by Corbusier and soon after, he became one of Corbusier's main assistants. In 1958, under the tutelage of Le Corbusier, Xenakis he designed the Philips Pavilion at International Expo 58 in Brussels. Expo 58, also known as the Brussels World's Fair, was held from 17 April to 19 October 1958 and it was the first major World Expo registered under the Bureau International des Expositions (BIE) after World War II. While the main ideas and sketch of the building was done by Le Corbusier, Xenakis did all the developments and the whole process of the project.[5]

Simultaneously with his successful career as a modern architect, Xenakis studied classical music with some of the famous teachers of the time: Nadia Boulanger, Arthur

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[3] Nicole V. Gagné, *Historical Dictionary of Modern and Contemporary Classical Music* (Lanham: Scarecrow Press, 2012), 299.
[4] Le Corbusier was a pioneer French architect and designer best known for the developments he made in modern architecture.
[5] Gagné, 2012, 305.

Honegger, and Darius Milhaud. All of them however, discouraged Xenakis from continuing his music because they didn't like the Xenakis' mathematical and structural approach to musical composition. These negative reactions were disappointing for him. Xenakis continued until he started taking lessons with Olivier Messiaen, who encouraged Xenakis to use his own background as an architect and mathematician as a deliberate influence in his process, incorporating complex mathematical formulas in his compositions.[6]Messiaen was an advocate for Xanakis to explore new approaches to composition. Messiaen later recalled:

> I understood straight away that he was not someone like the others. [...] He is of superior intelligence. [...] I did something horrible which I should do with no other student, for I think one should study harmony and counterpoint. But this was a man so much out of the ordinary that I said... No, you are almost thirty, you have the good fortune of being Greek, of being an architect and having studied special mathematics. Take advantage of these things. Do them in your music.[7]

Following Messieaen's advice, Xenakis wrote many of his pieces using his unique composition method of employing mathematic formulas and theories. Specific examples of mathematics, statistics, and physics applied to music composition can be found in Xenakis's use of the statistical mechanics of gases in *Pithoprakta*, statistical distribution of points on a plane in *Diamorphoses*, minimal constraints in *Achorripsis*, the normal

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[6] Nouritza Matossian, *Xenakis* (London: Kahn and Averill, 1986), 25–37.
[7] Matossian 1986, 43.

distribution in *ST/10* and *Atrées*, Markov chains in *Analogique*, game theory in *Duel*, *Stratégie*, and *Linaia-agon*, group theory in *Nomos Alpha*, set theory in *Herma* and *Eonta*.[8] His music can be considered an interdisciplinary partnership between music, math, geometry, architecture and mechanical engineering.

Philips Pavilion

In the process of designing the Philips Pavilion, Xenakis used nine hyperbolic paraboloid equations which each have slightly different values, basing based these floating, arch-like surfaces on Le Corbusier's original sketch. It may be useful at this point to define a hyperbolic paraboloid. "A hyperbolic paraboloid is an infinite surface in three dimensions with hyperbolic and parabolic cross-sections. A couple of ways to parameterize it and write an equation are as follows:"[9] $ZZ = \frac{yy^2}{bb^2} - \frac{xx^2}{aa^2}$

We use the term *hypar* to mean a hyperbolic paraboloid shape, or more formally a partial hyperbolic paraboloid, cut from the full infinite surface. The term *hypar* was introduced by the architect Heino Engel in his 1967 book *Structure Systems.*[10] *Hypars* are

[8] Ilias Chrissochoidis, Houliaras Stavros, and Christos Mitsakis, "Set theory in Xenakis' EONTA," In *International Symposium Iannis Xenakis*, ed. Anastasia Georgaki and Makis Solomos, Athens: The National and Kapodistrian University, 241–49.
[9] Erik Demaine, Martin Demaine, and Anna Lubiw, "Hyperbolic Paraboloids," *Erik Demaine's Folding and Unfolding*, 2014, http://erikdemaine.org/hypar.
[10] Heino Engel, *Structure System* (New York, Praeger, 1967), 215.

widely used in architecture designs. The Philips Pavilion at the 1958 Brussels exhibition is a beautiful surface, made of nine *hypars*, that rests on the ground. According to University of Oxford Math Institute, “Hyperbolic paraboloids are the canonical example of a surface with a "saddle point" - a stationary point which is neither a maximum nor a minimum”. At such points on surfaces, the Gaussian curvature is negative. Hyperbolic paraboloids are ruled surfaces. They also belong to the more specific family of "doubly ruled surfaces", alongside only hyperboloids of one sheet and (trivially) planes. Doubly ruled surfaces have two distinct straight lines through each point on them, and can be "swept out" by lines in two different ways.[11]

Image 1 provides a sketch of a typical hyperbolic paraboloid

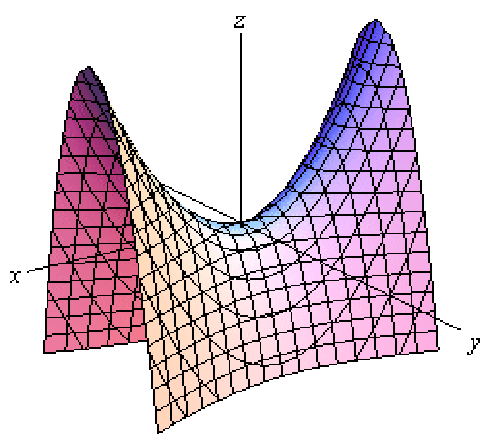

Image No. 1 - Hyperbolic paraboloid used in *Metastaseis* and the Philips Pavilion.[12]

[11] “Hyperbolic Paraboloids,” *Quadric Surfaces*, University of Oxford Math Institute, September 2015, https://www.maths.ox.ac.uk/about-us/departmental-art/quadric-surfaces/hyperbolic-paraboloids

[12] W. H Beyer, *CRC Standard Mathematical Tables*, 28th ed. Boca Raton (Florida: CRC Press, 1987) 227.

Now we can compare the surrounding space of a typical hyperbolic paraboloid to the first handwritten sketches of Philips Pavilion (See Image No. 2) and the first actual constructed idea of the pavilion, which used two simple hyperbolic paraboloids (See Image No. 3).

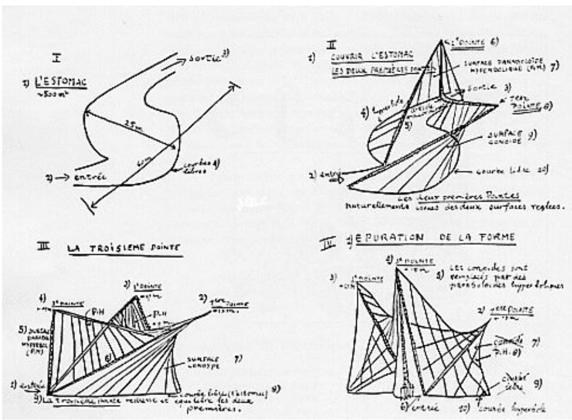

Image No. 2 - Hyperbolic paraboloid.[13]

Xenakis combined multiple hyperbolic paraboloids to achieve the modernistic structure of the Pavilion. His design of the building was encouraged by architects at the time. In Image No. 3, we can see the first actual, physical maquette (a sculptor's small preliminary model or sketch) of the building. As we can see here, the first sketch and this preliminary maquette used only two simple hyperbolic paraboloids.

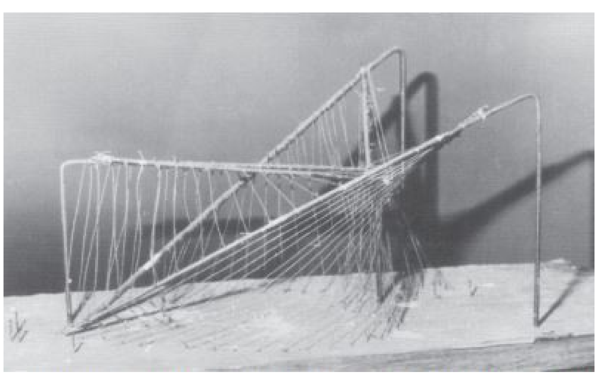

Image No. 3 - Primary sketch of *Philips Pavilion*, Le Corbusier, 1957, source: Le Corbusier Foundation, Paris.

[13] Matossian 1986, 110.

In Image No. 4, we see the final structure of Philips Pavilion in which Xenakis used nine complex hyperbolic paraboloids that became intertwined with each other.

Image No.4 – The final structure of the Philips Pavilion for International Expo 58, also demonstrating the hyperbolic paraboloids originally used in *Metastaseis*.

Metastaseis

While the Pavilion was under construction, Xenakis started to work on the relationship between geometry and music, and the possibility of a direct inspiration from geometric formulas on approaches to music composition.

He used the exact mathematical formulas as the Pavilion in one of his early prominent works, *Metastaseis* for orchestra, and fortunately several of Xenakis's handwritten sketches survive from the process of converting the special geometry used in the structure of the Philips Pavilion to the theoretical underpinnings of an orchestral work. *Metastaseis* was one of the first pieces composed by Xenakis in which architecture played a fundamental role.[14]

In the program note for *Metastaseis,* Xenakis explains the meaning of title of the piece: "*Meta*" means "after" or "beyond", and "*staseis"* or "immobility" refers to a state of standstills or extreme immobility. He explains that *Metastaseis* is a type of dialectic between classical music (referring to the use of serialism in the piece) and formulized music in which the composer is forced to obey the mathematic and rules of formulas. According to the program note, Xenakis had been more innovative in this particular piece than in his previous pieces in terms of orchestration, intervallic structure, and individual use of glissandi. In Metastaseis, Xenakis uses the orchestra in total divisi: 61 instruments play 61 individually distinct parts. With this innovative approach to voicing, Xenakis introduced the *mass conception* in music, in which music is built with a large number of sound events. Each of the string parts is given individual glissandi.

Formal Structure

The first element of *Metastaseis* that Xenakis incorporated from the mathematical concept of the hypar is its formal structure. As shown earlier in Image No. 1, a hypar has two curved sides on the left and right, which are called hyperbolics, and a deep empty valley in the middle, and its mirror underneath, which are called

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[14] Peter Hoffmann, "Xenakis, Iannis," *The New Grove Dictionary of Music and Musicians*, second edition, edited by Stanley Sadie and John Tyrrell (London: Macmillan Publishers, 2001).

paraboloids. Let's look at the figure that Xenakis used in his piece. If X and Y are replaced with numbers of 1 and -1 in the formula ($X^2 - Y^2 = Z$), the answers on coordinate system would be two ellipses (paraboloids) on the X and Y axes:

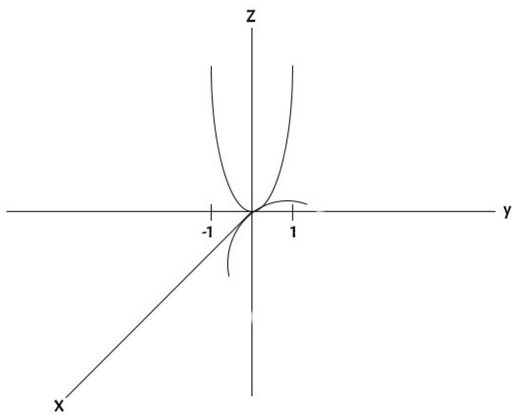

Image No. 5 – Paraboloids – diagram drawn by the author

In order to add hyperbolics to what we have above, we need to limit the possible answers of Z to two ranges of only more than zero ($X^2 - Y^2 = Z > 0$) and only less than zero ($X^2 - Y^2 = Z < 0$). If we draw answers and add them to the paraboloids of Image No. 5, the result would be:

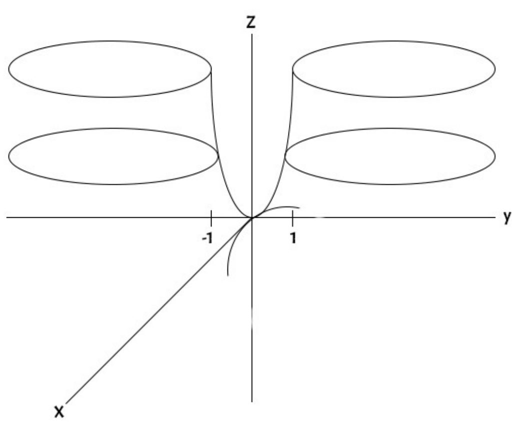

Image No. 6 – hyperbolic paraboloid on coordinate system – diagram drawn by the author

Now, if we just partially isolate the diagram and cut the side curves, the resulting image is as follows:

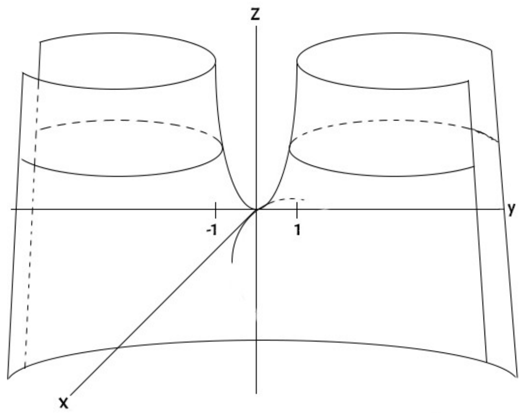

Image No. 7 – isolated hypar, cut sides – diagram drawn by the author

Xenakis drew a miniature compact version of his score which has nothing to do with the micro structure of the music, but it is a visualized version of the structure and what the piece *sounds like*.

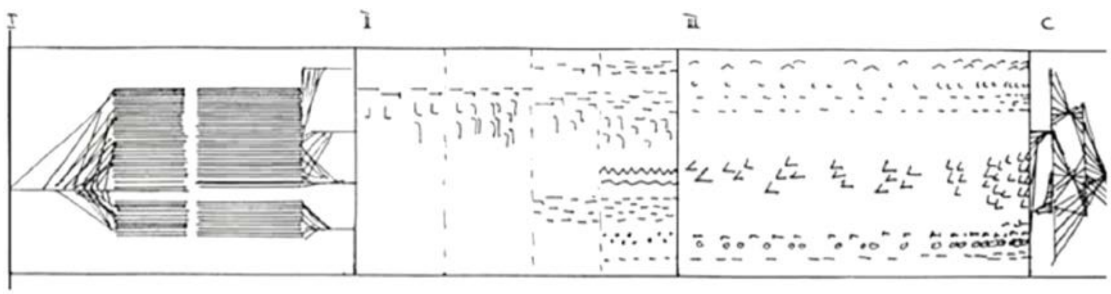

Image No. 8 – Xenakis's handwritten sketch of the *Metastaseis* Structure

The piece has four sections which are clear in the handwritten sketch above (Image No. 8). The structure of the form is similar to the hypar seen in Image No. 7: there are two curved hyperbolics on the sides and one valley in the middle.

Strings play the most important role in *Metastaseis*. Each individual instrument in the string section has its own separate part. The piece begins with a sustained single note, and listeners hear all string parts playing the same note, filling the atmosphere with an incredible sense of musical space. Gradually, string instruments begin to slide one by one with their individual, slow glissandi, which increases the tension more and more. All glissando notes began at that unison pitch and will slide up (in the violin and viola parts) and down (in cello and contrabass parts) to land on different pitches, resulting a heavy cluster of dissonant intervals. As we have seen, there are two mirrored curves on the left and right sides in hypars. What would be the best musical equivalent of a curved line? Xenakis's answer is glissandi, since cumulatively, they have the transformational quality of curved lines in comparison to non-vibrato sounds and straight lines. The next sections of this paper will examine these glissandi in more detail.

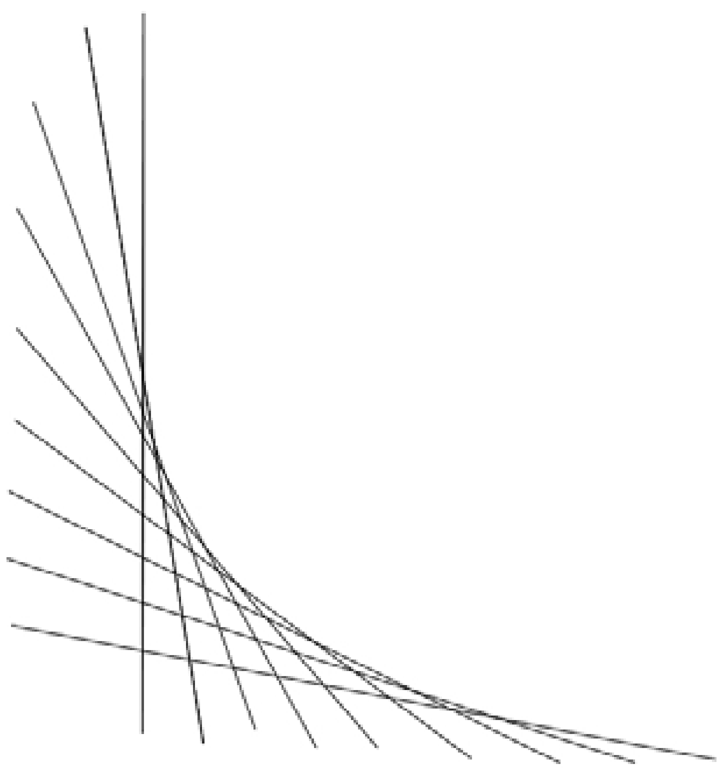

Image No. 9 – making a curved edge with straight lines – diagram drawn by the author

The First Glissando Section (bars 1-35)

In the beginning of the piece, the string instruments divide into two sections: the first section is made up of 24 violins and eight violas, which draw a curved edge with their ascending glissando, and the section is comprised of cellos and contrabasses, which in their downward motion create a mirrored curve. In the first bar, the entire string section plays a unison G♮, and the first individual violin immediately starts a very slow ascending glissando which spans a diminished 5th from G♮ to D# across eight bars, and then expands another diminished 6th to D in the next seven bars. This is the uppermost violin line. All other 23 violins and eight violas also begin their ascending glissandos during the first 35 bars of the piece (see Image No. 10). In contrast to the slow upward motion of the highest strings, the lower members of the violin and viola sections have rapid glissandi that span much larger intervals. For example, the first viola has a slow and slight glissando which begins in bars 13-15, moving from G to A, then moving a perfect 5th in bars 16-24 from A to D. However, the eighth and last viola player begins their glissando in bar 32, starting on G and quickly passing two octaves to arrive on C, thus spanning an 11th across only 2 and a half bars. This contrast, with upper lines sliding up slightly and lower lines moving quickly to larger intervals, creates the effect of hyperbolic paraboloids, wherein a curved surface is made by slightly sloped straight lines. In Image No. 10, we see that all 24 violins and eight violas start on unison, then glissando to higher notes in this fashion.

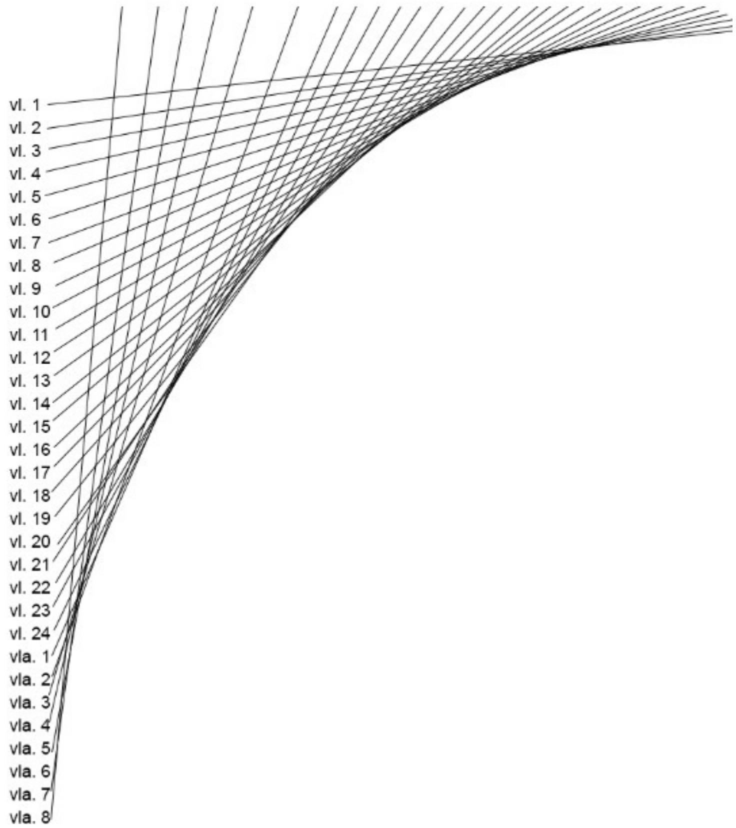

Image No. 10 – violins and violas in the first 35 bars – diagram drawn by the author

The lower section of strings (cellos and contrabasses) demonstrates a similar, mirrored motion, with the uppermost instruments descending slightly and slowly beginning in bar 18, while the contrabasses play faster and deeper glissandi. Image No. 11 shows the inner semicircular built by the outer surfaces of the hyperbolic paraboloid structure across the first 35 bars of *Metastaseis*. Image No. 12 shows the score of bars 1-35. Together, the upper and lower string sections give the sense that the orchestra is stretching as the range of their respective glissandi expand throughout the piece. In this portion of the work, the hypar is denser in its upper half because twice as many string parts are given ascending glissandi.

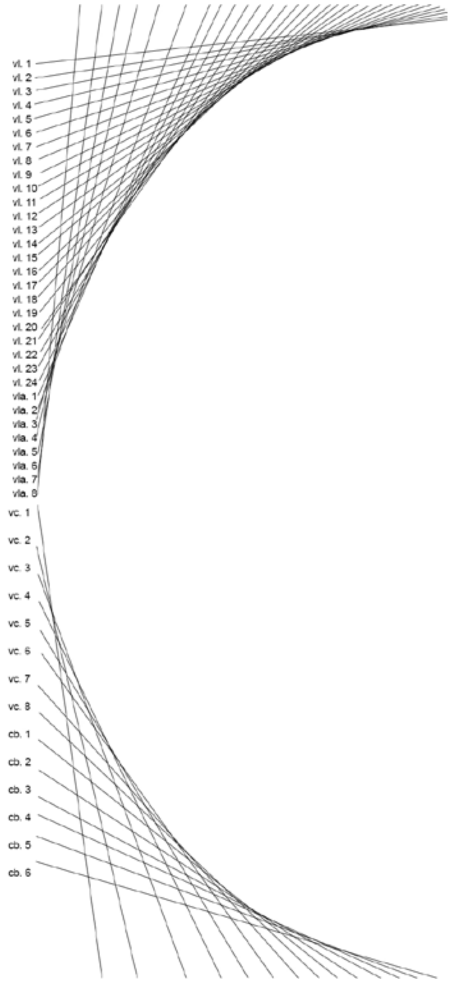

Image No. 11 – hypar built by glissandos, bars 1-35 – diagram drawn by the author

This first section is similar to an ABA' form because, after the 52-bar expansion effect created by these glissandi, the section concludes by returning to a single unison pitch, this time rather than the original G. In his book *Xenakis: His Life in Music* James Harley explains Xenakis's use of glissandi in *Metastaseis*:

> Glissandi were nothing new, of course. The portamento had been commonly used to add a certain sentimental expression, as in the work of Gustav Mahler, one of the first to notate the effect explicitly. Bela Bartok abstracted the technique much farther, and was no doubt an influence, along with

Edgard Varèse's sirens in *Ionisation* (1931), perhaps. Xenakis treated the glissando as an independent sonic entity, creating a musical space in which a transition from a single pitch to a forty-six-note cluster is achieved by means of a continuous evolution of sound.

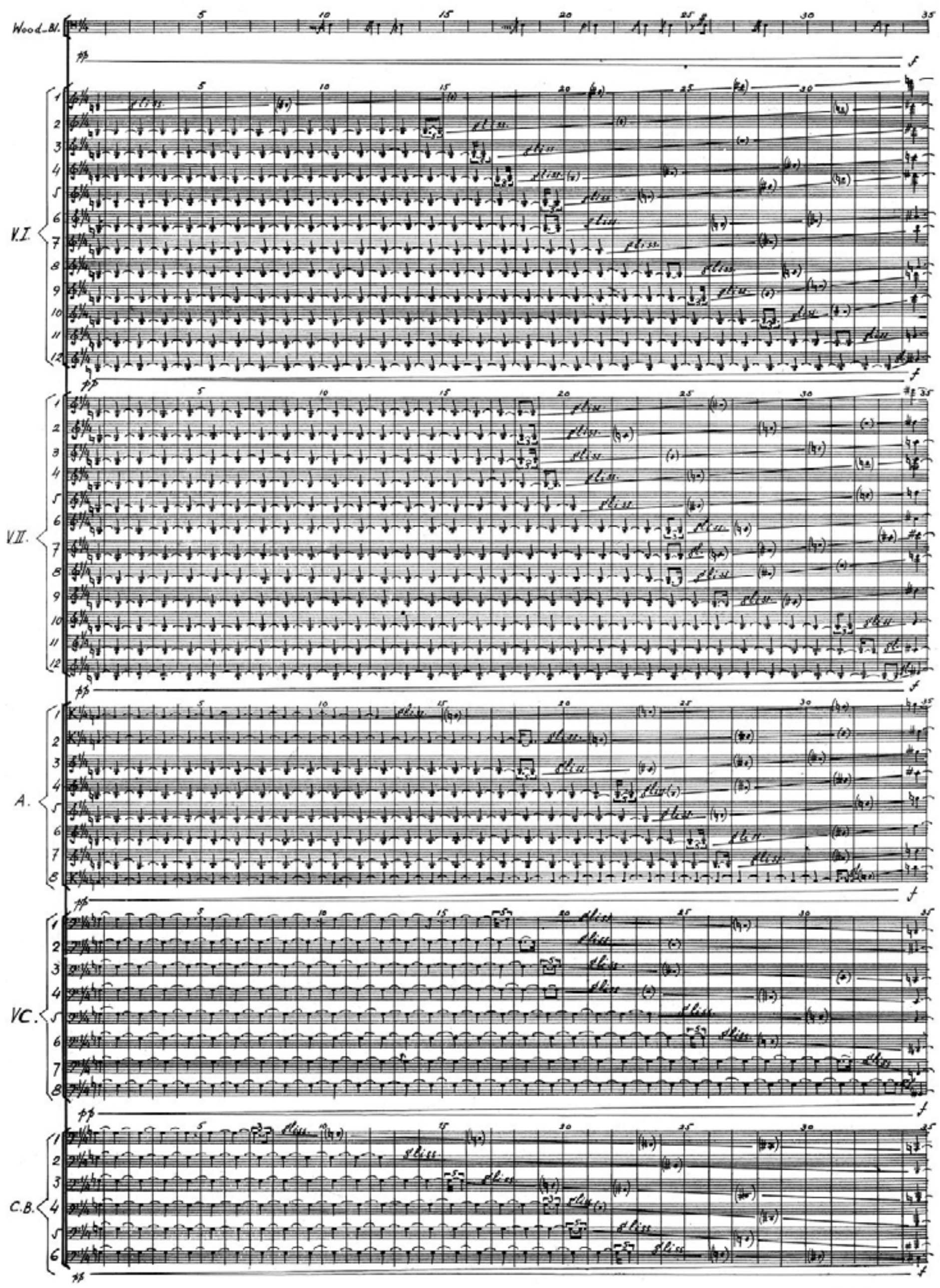

Image No. 12 – first page of the score of *Metastaseis* [15]

[15] Iannis Xenakis, *Metastaseis* (London; New York: Boosey & Hawkes, 1967).

The Last Glissando Section (bars 305-345)

As mentioned earlier, the first and last sections of *Metastaseis* are intended to build hyperbolic paraboloids using string glissandi as the principal tool. In the first section (bars 1-35) we see glissandos expanding from the unison opening, and in the last section (bars 305-345) we see cumulative glissandi ending on unison at the conclusion of the piece. Image No. 13 shows Xenakis's handwritten sketch of this last section, which shows that the parabolic sonic effects employed here are more complex than in the first section. Here, the upper violins slide downward while the other three parts are given upward motion at different rates, the lower violins sliding up quickly, the cellos ascending slowly, and the contrabasses sliding to higher pitches with a sharp grade.

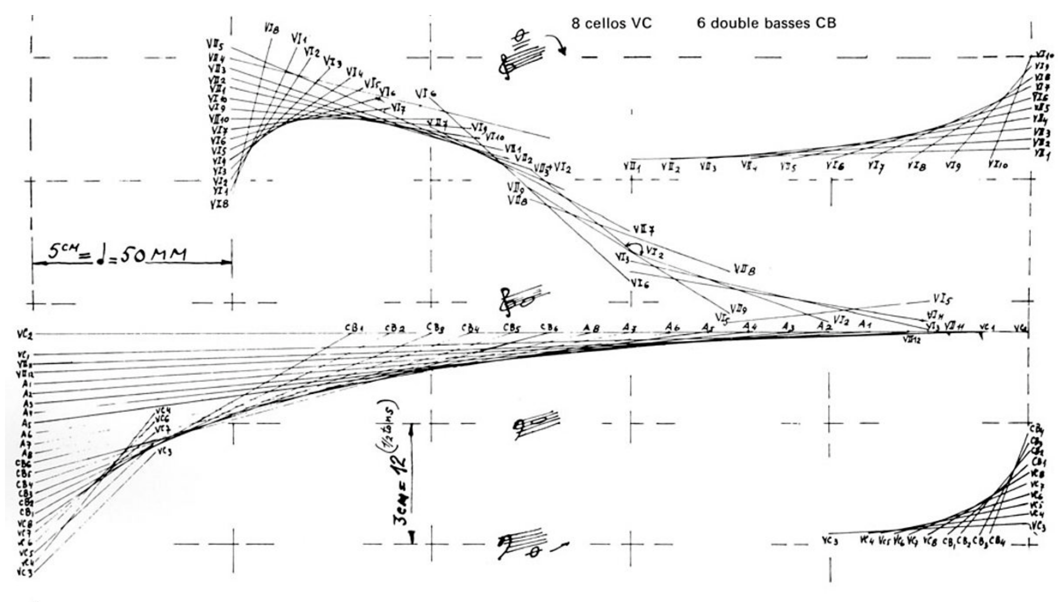

Image No. 13 – sketch of *Metastaseis*.[16]

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[16] Varga 1996, 74.

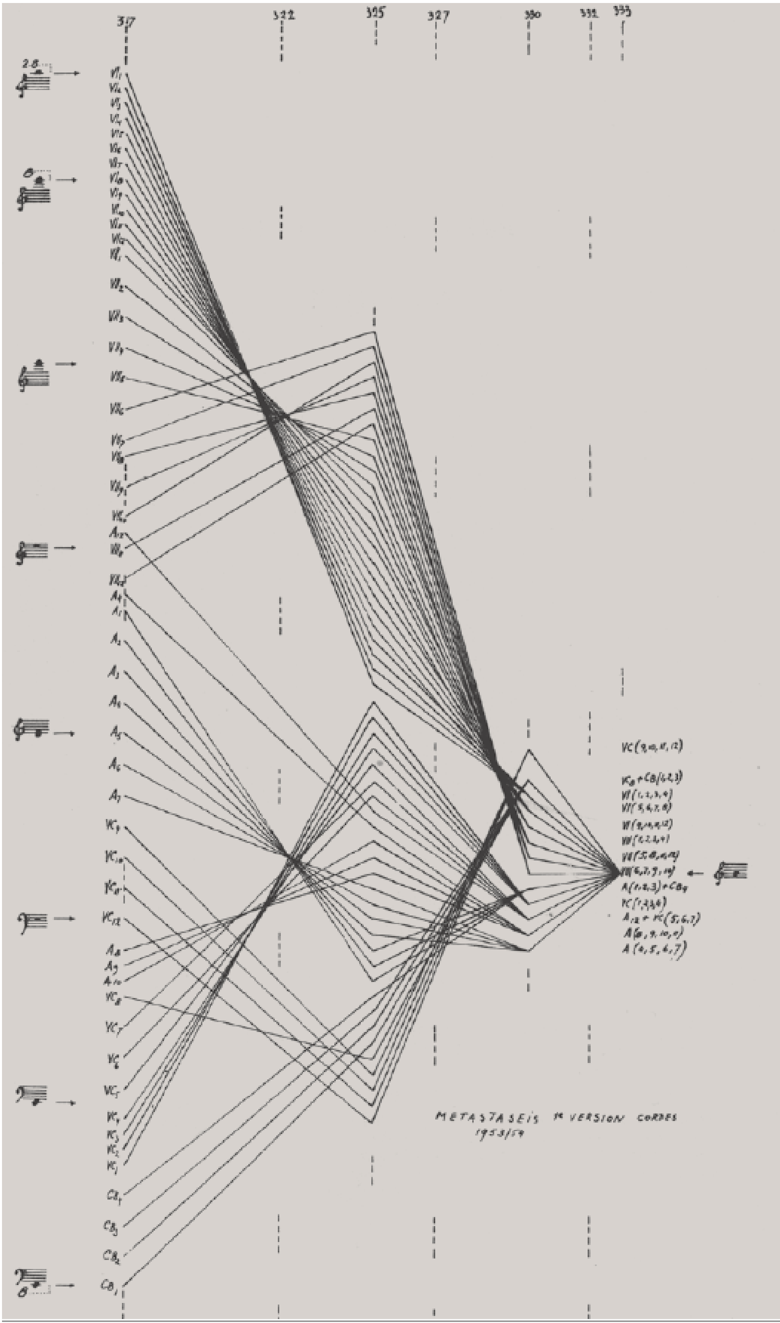

Image No. 14 – Graph plotting paths of strings glissandi in *Metastaseis*: bars 317-333, source: Matossian 1986, 62.

Meanwhile, Image No. 14 is Xenakis's sketch for the ending of the piece, which shows how he used arch-like hyperbolic paraboloids to achieve the effect of reaching to a unison sound from 40-tone cluster in several stages.

Pitch Material in the Glissando Sections

reason that all glissandi should begin or end up in unison is the fact that hyperbolic paraboloids are built around the center of coordinate system, and the empty steep valley of the middle (no matter how small or large it is) is built around the zero point of XYZ (0, 0, 0). In the pitch calculation for this piece, the destinations—these unison pitches—are the most important structural locations. The pitches between the opening unison and the final destination of the glissando sections could be measured and divided by the number of bars in order to calculate a given string part's approximate pitch according to the degree of their individual slope.

Xenakis selected G for the unison at the beginning of the piece because G is an open string in all string instruments, and also it gave Xenakis more space from the lowest note in the violin part in order to maximize their ascending glissandos. Thus G functions as the zero point of this hyperbolic paraboloid. Recall the three-dimensional figure, graphed on a coordinate system, found in Image No. 7. Musically-speaking, numbers are computed in semitones, so as an example, the coordinates (0, 0, 1) represent one semitone more or less than G♮, or G#/Ab or F#/Gb. (Note that numbers in the XYZ locations are not necessarily measured by any conventional measurement system, such as meters or inches, but are proportional indicators in reference to the point zero. Calculating the exact points needed to be accomplished by a computer; Xenakis employed an IBM 7090 computer, a computer designed specifically for the composer. This info is mentioned at the beginning of the score.

XYZ locations could be assigned to pitches. According to the hyperbolic paraboloid formula, I have calculated the first four points of the first hypar in which Z>0.The second point on the positive paraboloid after point zero of XYZ (0, 0, 0) is (1, 1, 0), which represents two semitones (the sum of

1 and 1) and would be sounded as A♮, a major 2nd above G♮. This motion appears in the score in the first viola part (bar 13) and second viola part (bar 19), who begin a glissando from G♮ to A♮. Below I have traced the first four positive spots:

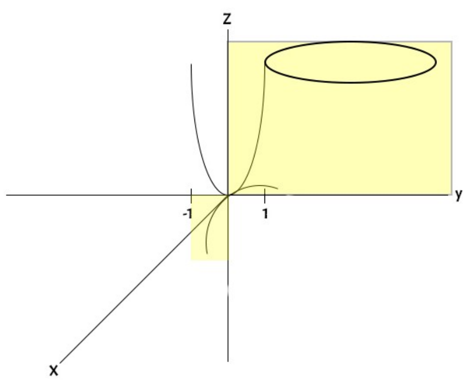

Image No. 15 – positive part of a hyperbolic paraboloid – diagram drawn by the author

1- Zero point, XYZ (0, 0, 0), unison G♮ in the beginning of the piece (Image No. 12)

2- XYZ (1, 0, 1), 2 positive semitones (ascending major 2nd), A♮ in relation to G♮. Viola no. 1 bar, 13, and viola no. 2, bar 19

2- XYZ (1, 1, 2), 4 positive semitones (ascending major 3rd), B♮ in relation to G♮. Violin I no. 4, bar 18, and viola 4, bar 23

3- XYZ (1, 1, 3), 5 positive semitones (ascending perfect 4th), C♮ in relation to G♮ . Violin I no. 8, bar 25; violin II no. 2, bar 19; violin II no. 6, bar 25

4- XYZ (1, 1, 3), 6 positive semitones (ascending augmented 4th), C# in relation to G♮. Violin II no. 9, bar 27; violin II no. 11, bar 33; viola no. 3, bar 19

Three rules in computing the locations and assigning them to pitches that should be taken into consideration:

5- The order of arrival points is different from the order in which string instruments begin their glissandi. For example, the first glissando begins in violin I no. 1, but its arrival on D# occurs in bar 9, after the first non-zero point location on by in bar.

6- Each point should be bigger than the previous point location, for example, the firstnon-zero point is (1, 0, 1), so the next points always have an X and Z larger than 1.

7- Since a hypar has two positive and negative sides, which don't precisely match, there are a series of points on the left side of the hypar diagram (locations not yellowed in Image No. 15) where the sum of the XYZ numbers would be a negative integral. This part is even more difficult to compute since both positive and negative numbers are found in the points' locations (for example XYZ (-1, 3, -4), which must be computed with advanced machines in order to achieve correct order). Of the order of point locations are not correct, the final result on the coordinate system would not align on the original hyperbolic paraboloid.

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